This question paper has 9 printed pages

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BAL BHARATI PUBLIC SCHOOL
SAMPLE TEST PAPER - (2020-21)
CLASS – XII
SUBJECT: MATHEMATICS

M.M: 80 **TIME: 3 Hrs**

GENERAL INSTRUCTIONS

1. *All questions are compulsory.*
2. *The question paper consists of 38 questions divided into five sections I, II, III, IV & V.*
3. *Section I comprises of 16 very short answer type questions of 01 mark each,*
   Section II comprises of 2 case studies. Each case study comprises of 5 case based MCQs .
   An examinee is to attempt any 4 out of 5 MCQs.
   Section III comprises of 10 questions of 02 marks each
   Section IV comprises of 7 questions of 3 marks each.
   Section V comprises of 3 questions of 5 marks each.
4. *There is no overall choice. However internal choice has been provided in three questions of Section III, two questions of Section IV and three questions of Section V. You have to attempt only one of the alternatives in all such questions.*

PART A

SECTION I

ALL QUESTIONS ARE COMPULSORY. IN CASE OF INTERNAL CHOICES, ATTEMPT ANY ONE.

1) The diagonals of a parallelogram are represented by the vectors $\overrightarrow{d_1} = 3\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\overrightarrow{d_2} = \hat{\imath} - 3\hat{\jmath} + 4\hat{k}$. Find the area of the parallelogram.

2) If A = {1,2,3} and let R= {(1,1), (2,2),(3,3),(1,2),(2,1),(2,3),(3,2)}. Check whether the given relation is symmetric and transitive or not.

OR

Consider a function f: Z → Z defined as f(x) = x^2. Check the injectivity and surjectivity of f.

3) Evaluate $\int_{-1}^{1} x^{17} \cos^4 x \, dx$

OR

$\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx$

4) Three integers are chosen at random from first 20 integers. Find the probability that their product is even.

5) If a line makes α, β and γ with the x, y and z axes respectively, then find the value of $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$.

6) A relation R in the set of real numbers **R** defined as R = {(a,b) : $\sqrt{a}$ = b} is a function or not. Justify.

OR

An equivalence relation R in A divides it into equivalence classes A_1, A_2, A_3. What is the value of $A_1 \cup A_2 \cup A_3$ and $A_1 \cap A_2 \cap A_3$.

7) Find the restriction on p and k so that PY+WY is well defined, where the matrices Y, W and P are of the order 3 x k , 2 x 3 and p x k respectively.

8) If A is a square matrix of order 2 and $|adj\ A|$ = 3, then, find the value of $|2AA'|$.

9) Find the area bounded by the curve y= sin2x, x axis and the lines x = $\pi/4$ and $3\pi/4$.

10) Find the projection of $\vec{a} = 3\hat{\imath} - \hat{\jmath} - 2\hat{k}$ on $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$.

11) If A and B are events such that P(A) = ½ ,P(B) = 7/12 and P(A' ∪ B') = ¼, state whether A and B are mutually exclusive or independent.

12) Find the sum of the degree and the order of the differential equation, $\frac{d}{dx}[(\frac{d^2y}{dx^2})^4] = 0$.

13) If $|\vec{a} - \vec{b}| = |\vec{a} + \vec{b}|$, *then* find the relation between vectors a and b.

OR

If the vectors $\vec{a} = 4\hat{\imath} + 11\hat{\jmath} + x\hat{k}$, $\vec{b} = 7\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$ and $\vec{c} = \hat{\imath} + 5\hat{\jmath} + 4\hat{k}$ are coplanar, then find the value of x.

14) If A and B are the square matrices of order 3 each, $|A| = 2$ and $|B| = 3$, then find the value of $|3AB^{-1}|$.

OR

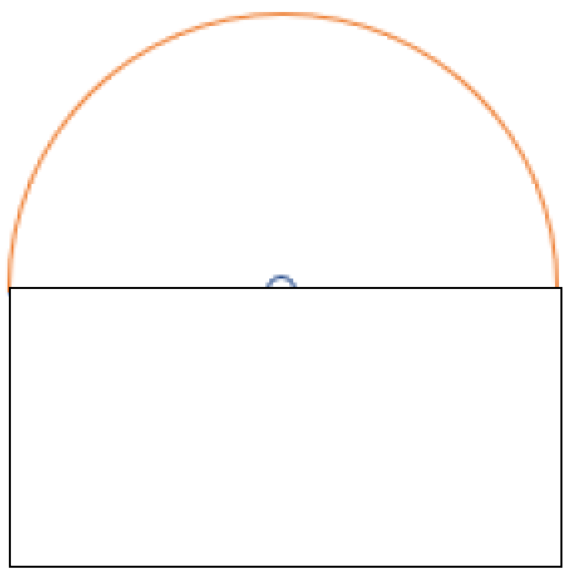
Find the value of k for which the set of equations x+ ky+3z = 0, 3x+ky-2z=0 and 2x+3y-4z=0 has a unique solution.

15) Let A={1,2,3,4}. Let R be the equivalence relation on A x A defined by (a,b) R (c,d) iff a+d = b+c. Find the equivalence class [(1,3)].

16) Find the point where the line $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z+5}{4}$ meets the plane 2x + 4y-z = 3.

SECTION II

17) Mr Shashi, who is an architect, designs a building for a small company. The design of window on the ground floor is proposed to be different than other floors. The window is in the shape of a rectangle which is surmounted by a semi-circular opening. This window is having a perimeter of 10 m as shown below



Based on the above information answer the following:

(i) If 2x and 2y represent the length and the breadth of the rectangular portion of the window, then the relation between the variables is given by

(a) $4y - 2x = 10 - \pi$

(b) $4y = 10 - (2 - \pi)x$

(c) $4y = 10 - (2 + \pi)x$

(d) $4y - 2x = 10 + \pi$

(ii) The combined area (A) of the rectangular region and semi-circular region of the window expressed as a function of x is

(a) A =10x+(2+ $\frac{1}{2}$ π) x^2

(b) A =10x-(2+ $\frac{1}{2}$ π) x^2

(c) A =10x-(2- $\frac{1}{2}$ π) x^2

(d) A= 4xy+ $\frac{1}{2}$ π x^2

(iii) The maximum value of area of the whole window, A is

(a) A= $\frac{50}{4+\pi}$ cm^2

(b) A= $\frac{50}{4+\pi}$ m^2

(c) A= $\frac{100}{4+\pi}$ m^2

(d) A= $\frac{50}{4-\pi}$ m^2

(iv) The owner of this small company is interested in maximizing the area of the whole window so that maximum light input is possible.
For this to happen, the length of rectangular portion of the window should be

(a) $\frac{20}{4+\pi}$ m

(b) $\frac{10}{4+\pi}$ m

(c) $\frac{4}{10+\pi}$ m

(d) $\frac{100}{4+\pi}$ m

(v) In order to get the maximum light input through the whole window, the area (in sq. m) of the semi-circular opening of the window is

(a) $\frac{100}{(4+\pi)^2}$

(b) $\frac{50\pi}{4+\pi}$

(c) $\frac{50\pi}{(\pi+4)^2}$

(d) same as the area of rectangular portion of the window

18) There are three categories of students in a class of 60 students:

A : Very hard working students
B : Regular but not so hard working
C : Careless and irregular.

It’s known that 10 students are in category A, 30 in category B and rest in category C.

It is also found that probability of students of category A, unable to get good marks in the final year examination is, 0.002, of category B it is 0.02 and of category C, this probability is 0.20.

Based on the above information answer the following:

(i) If a student selected at random was found to be the one who could not get good marks in the examination, then the probability that this student is of category C is

(a) $\frac{201}{231}$

(b) $\frac{200}{231}$

(c) $\frac{31}{231}$

(d) $\frac{21}{231}$

(ii) Assume that a student selected at random was found to be the one who could not get good marks in the examination. Then the probability that this student is either of category A or of category B is

(a) $\frac{31}{231}$

(b) $\frac{200}{231}$

(c) $\frac{201}{231}$

(d) $\frac{21}{231}$

(iii) The probability that the student is unable to get good marks in the examination is

(a) $\frac{231}{300}$

(b) $\frac{231}{3000}$

(c) $\frac{770}{1000}$

(d) 0.007

(iv) A student selected at random was found to be the one who could not get good marks in the examination. The probability that this student is of category A is

(a) $\frac{1}{231}$

(b) $\frac{200}{231}$

(c) $\frac{230}{231}$

(d) None of these

(v) A student selected at random was found to be the one who could not get good marks in the examination. The probability that this student is **NOT** of category A is

(a) 0

(b) $\frac{230}{231}$

(c) $\frac{21}{231}$

(d) 1

<u>SECTION III</u>

19) Let tan^{-1}x + tan^{-1}y = $\frac{\pi}{4}$, then find the value of x + y +xy.

20) Evaluate

$\int \frac{\sqrt{tanx}}{sinx\ cosx}$ dx OR $\int_0^{\pi/4} \log(1 + tanx)\ dx$

21) If $\vec{a}$ and $\overrightarrow{b}$ are the unit vectors, then what is the angle between $\vec{a}$ and $\overrightarrow{b}$ for $\vec{a}$ -$\sqrt{2}$ $\overrightarrow{b}$ to be a unit vector?

22) Two cards are drawn successively without replacement, from a well shuffled pack of 52 cards. Find the probability distribution of the number of kings drawn.

23) Sketch the region: $\{(x, y): x^2 + y^2 \leq 1 \leq x + y\}$

24) Find the general solution of the given differential equation:
(1+x^2)dy + 2xy dx = cotx dx (x≠ 0)

25) If X$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ = $\begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$, then find the matrix X.

OR

Show that all the diagonal elements of a skew symmetric matrix are zero.

26) Find a unit vector perpendicular to each of the vector $\vec{a}$ + $\vec{b}$ and $\vec{a}$ - $\vec{b}$, where $\vec{a}$ = 3$\hat{\imath}$ + 2$\hat{\jmath}$ + 2$\hat{k}$ and $\vec{b}$ = $\hat{\imath}$ + 2$\hat{\jmath}$ - 2$\hat{k}$.

27) Find the value of a for which the function 'f' defined as

f(x) = $\begin{cases} a \text{ , } when\ x = 0 \\ \frac{tanx - sinx}{x^3} \text{ , } x \neq 0 \end{cases}$ is continuous at x = 0.

28) Find the condition for the curves $\frac{x^2}{a^2}$ - $\frac{y^2}{b^2}$ = 1 , xy = c^2 to intersect orthogonally.

SECTION IV

29) Solve the following D.E.

[y – x cos ($\frac{y}{x}$)] dy + [y cos ($\frac{y}{x}$) – 2x sin ($\frac{y}{x}$)] dx = 0

30) Find the integral:

$$\int \frac{dx}{x(x^5 + 3)}$$

31) Using integrals, find the area of the ellipse 9x^2 + 4y^2 = 36

OR

Using Integrals, find the area of the region bounded by the parabola y^2 = 4ax and its latus rectum.

32) If x = $\sqrt{a^{sin^{-1}t}}$, y = $\sqrt{a^{cos^{-1}t}}$, show that $\frac{dy}{dx}$ = -$\frac{y}{x}$

OR

Prove that the greatest integer function defined by f(x) = [x], 0 < x < 2 is not differentiable at x=1.

33) Find the intervals in which the function f(x) = 3x^4 - 4x^3 - 12x^2 + 5 is

(i) Strictly increasing (ii) strictly decreasing

34) Let A ={ x ∈ Z : 0≤ x ≤ 15}. Show that R = {(a,b)∈ A, $|a - b|$ is a multiple of 7} is an equivalence class. Also write the equivalence class [2].

35) Find $\frac{dy}{dx}$, where y = cos^{-1}(4x^3 - 3x) , if -$\frac{1}{2}$ < x < $\frac{1}{2}$

SECTION V

36) Solve the following LPP graphically

Max Z=7.8x+7.1y

Sub to

$3x + 4y \leq 1080$

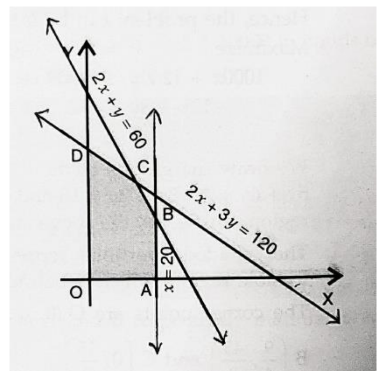
$3x + 8y \leq 1920$

$x \leq 200$

$x \geq 0, y \geq 0$

OR

Observe the given region and answer the following parts:



i). Identify and write the constraints to which the Objective function is subjected .

ii) If the Objective Function is $Z = 7.5\,x + 5y$, find its maximum value and the point at which it is maximum.

37) If A = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and B= $\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$ find the product AB and use this result to solve the following system of linear equations

$2x - y + z = -1$

$-x + 2y - z = 4$

$x - y + 2z = -3$

OR

If A = $\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the system of equations

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2$

$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5$

$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4$

38) Find the equation of the plane which contains the line of intersection of the planes $\vec{r} \cdot (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) - 4 = 0$, $\vec{r} \cdot (2\hat{\imath} + \hat{\jmath} - \hat{k}) + 5 = 0$ and which is perpendicular to the plane $\vec{r} \cdot (5\hat{\imath} + 3\hat{\jmath} - 6\hat{k}) + 8 = 0$.

OR

Find the distance of the point (-2, 3,-4) from the plane 4x +12y -3z -9= 0 measured parallel to the line

$$\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$$