BAL BHARATI PUBLIC SCHOOL

SAMPLE QUESTION PAPER-(2020-21)

CLASS XII MATHEMATICS

ANSWER KEY

SECTION A (1 MARK FOR THE CORRECT ANSWER, ZERO OTHERWISE)

1) $5\sqrt{3}$ square units 2) Symmetric but not transitive OR Neither injective nor surjective 3) 0 OR log$\vert x^{10} + 10^{x}\vert$ + c 4) 17/19 5) 2 6) R is not a function as $\sqrt{a}$ is not defined for negative real numbers. OR $A_1 \cup A_2 \cup A_3 = \text{A}$ and $A_1 \cap A_2 \cap A_3 = \emptyset$ 7) p = 2, k= 3 8) 36 9) 1 square unit 10) $\sqrt{14}$/2 11) $Neither\ M.E.nor$ independent. 12) 4 13) $\vec{a} \perp \vec{b}$ OR x = 10. 14) 18 OR k $\neq \frac{33}{2}$ 15) { (1,3) , (2,4)} 16) (3,-1,-1)	
SECTION II(1 MARK FOR THE CORRECT ANSWER, ZERO OTHERWISE) 17) (i) c	

(ii) d	
(iii) b	
(iv) a	
(v) c	
(ANY FOUR , OUT OF 5 , TO BE ATTEMPTED)	
18) (i) b	
(ii) a	
(iii) b	
(iv) a	
(v) b	
SECTION III	
19) Let tan^{-1}x = A and tan^{-1}y = B	½
=> x = tan A and y = tan B	
ATQ, A + B = $\pi/4$	
⇨ tan(A + B) = tan $\pi/4$	
=> $\frac{tanA+tanB}{1-tanAtanB}$ = 1	
=>$\frac{x+y}{1-xy}$ = 1	1
=> $x + y = 1 - xy$	½
=> x+y+xy = 1	
20). $\int \frac{\sqrt{tanx}}{sinxcosx}$ dx	
= $\int \frac{\sqrt{tanx}\, sec^2x}{sinxcosxsec^2x}$ dx (dividing num and den by cos^2x)	½
= $\int \frac{\sqrt{tanx}}{tanx} sec^2x\, dx$	

$= \int \frac{sec^2x}{\sqrt{tanx}}$ dx Put tanx = t , sec^2xdx = dt $\int \frac{dt}{\sqrt{t}}$ $2t^{1/2}$ + c = $2\sqrt{tanx}$ + c OR Let $I = \int_0^{\frac{\pi}{4}} \log(1 + tanx)\, dx$ $I = \int_0^{\pi/4} \log\left(1 + \tan\left(\frac{\pi}{4} - x\right)\right) dx$ So $I = \int_0^{\pi/4} \log\left(\frac{2}{1+\tan x}\right) dx = \int_0^{\pi/4} \{\log 2 - \log(1 + \tan x)\}\, dx$ Hence $I = (\log 2) \int_0^{\pi/4} dx - I$ which gives $2I = \pi(\log 2)/4$ Therefore $I = \frac{\pi}{8} \log 2$	½ ½ ½ ½ ½ ½ ½
21) Given $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert = 1$ Also $\lvert\vec{a} - \sqrt{2}\,\vec{b}\rvert^2 = (\vec{a} - \sqrt{2}\,\vec{b}) . (\vec{a} - \sqrt{2}\,\vec{b}) = 1$ ⇨ $\lvert\vec{a}\rvert^2 - 2\sqrt{2}\,\vec{a} . \vec{b} + 2\lvert\vec{b}\rvert^2 = 1$ On solving $\vec{a} . \vec{b} = \frac{1}{\sqrt{2}}$ $\lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\theta = \frac{1}{\sqrt{2}}$ $cos\theta = \frac{1}{\sqrt{2}}$ $\theta = \pi/4$	1 ½ ½
22) Let X denotes the number of kings in a draw of two cards. X = 0, 1, 2 X: 0, 1, 2 P(X): 188/221, 32/221, 1/221	½ 1½

X	0	1	2
P(X)	188/221	32/221	1/221

23) Correct Circle Correct line Correct region	½ ½ 1
24) $(1+x^2)dy + 2xy\, dx = \cot x\, dx$ $\Rightarrow \frac{dy}{dx} + \frac{2x}{1+x^2} y = \frac{\cot x}{1+x^2}$ $\frac{1}{2}$ I.F. $= e^{\int \frac{2x}{1+x^2}dx}$ $= 1+ x^2$ $\frac{1}{2}$ Sol: $y(1+ x^2) = \int \frac{\cot x}{1+x^2}(1+ x^2)\, dx$ ⇨ $y(1+ x^2) = \int \cot x\, dx$ ⇨ $y(1+ x^2) = \log\lvert \sin x\rvert + c$ 1	
25) Since the product of the matrices is a 2 x 3 matrix and the post multiplier of X is 2 x 3, so X is a 2 x 2 matrix. Let X = $\begin{bmatrix} a & b \\ x & y \end{bmatrix}$ Then the given equation becomes $\begin{bmatrix} a & b \\ x & y \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$ On solving X= $\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$ OR Let $A=[a_{ij}]$, be a square matrix such that it is skew symmetric Hence, $A = -A^T$ Therefore, $a_{ij} = -a_{ij}$ for all elements, For diagonal elements: i = j, so $a_{ii} = -a_{ii}$ which gives $2a_{ii} = 0$ or $a_{ii} = 0$	$\frac{1}{2}$ 1 $\frac{1}{2}$ ½ ½ 1
26) We have $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$. So $\vec{a} + \vec{b} = 4\hat{\imath} + 4\hat{\jmath} = (\vec{p}$ say) $\vec{a} - \vec{b} = 2\hat{\imath} + 4\hat{k}$ ($\vec{q}$ say) Now $\vec{p} \times \vec{q}$ / $\vec{q} \times \vec{p} = \pm(16\hat{\imath} - 16\hat{\jmath} - 8\hat{k})$ is the vector perpendicular to both $\vec{p}$ and $\vec{q}$ Unit normal vector $= \pm \frac{16}{24}\hat{\imath} \mp \frac{16}{24}\hat{\jmath} \mp \frac{8}{24}\hat{k}$	$\frac{1}{2}$ 1 $\frac{1}{2}$
27) Consider $\lim_{x\to 0} \frac{\tan x - \sin x}{x^3}$ $= \lim_{x\to 0} \frac{\sin x(1-\cos x)}{\cos x . x^3}$	

$= \lim_{x \to 0} \left(\frac{sinx}{x}\right) \left(\frac{1-cosx}{x^2}\right)\left(\frac{1}{cosx}\right)$ $= (1)(\frac{1}{2})(1)$ $= \frac{1}{2}$ As the function is continuous at x= 0, therefore, $\lim_{x \to 0}$ f(x) = f(0) ⇨ a=1/2.	$\frac{1}{2}$ 1 $\frac{1}{2}$
28) As the two curves intersect each other orthogonally, the tangents to the two curves at the point of intersection are perpendicular to each other. Given equations are $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, xy = c^2 Now, by differentiating these two equations $\frac{2x}{a^2} - \frac{2yy'}{b^2} = 0$ and y+x$\frac{dy}{dx}$= 0 1 Now $m_1 = \frac{xb^2}{ya^2}$ and m_2 = -y/x $\frac{1}{2}$ Now to intersect orthogonally, $m_1 m_2 = -1$ $\left(\frac{xb^2}{ya^2}\right)$ (-y/x) = -1 $\frac{1}{2}$ Or $b^2/a^2 = 1$ or, $b^2=a^2$ is the required condition.	
SECTION IV 29) $[y - x\cos(\frac{y}{x})]\, dy + [y\cos(\frac{y}{x}) - 2x\sin(\frac{y}{x})]\, dx = 0$ Put y = vx Differentiating w.r.t x, we get $\frac{dy}{dx} = v + x\frac{dv}{dx}$ We get $v + x\frac{dv}{dx} = \frac{2sinv - vcosv}{v - cosv}$ ⇨ $x\frac{dv}{dx} = \frac{2sinv - vcosv}{v - cosv} - v$ ⇨ $x\frac{dv}{dx} = \frac{2sinv - vcosv - v^2 + vcosv}{v - cosv}$	$\frac{1}{2}$ $\frac{1}{2}$

⇨ $x \frac{dv}{dx} = \frac{2sinv - v^2}{v - cosv}$ ⇨ $\int \frac{v - cosv}{2sinv - v^2} dv = \int \frac{dx}{x}$ (put $2sinv - v^2$ = t) ⇨ $\int \frac{-dt}{2t} = \int \frac{dx}{x}$ ⇨ $-\frac{1}{2}\log\left\|2\sin\left(\frac{y}{x}\right) - \left(\frac{y}{x}\right)^2\right\| = \log\|cx\|$	1 $\frac{1}{2}$ $\frac{1}{2}$
30) Consider $$\int \frac{dx}{x(x^5 + 3)}$$ Put x^5 + 3 = t, then $5x^4$dx = dt => dx = $\frac{dt}{5x^4}$ Now , we have $\frac{1}{5}\int \frac{dt}{(t-3)t}$ Using Partial Fractions, we get $\frac{1}{15}\int\left(\frac{1}{t-3} - \frac{1}{t}\right)dt$ = $\frac{1}{15}[log\|x^5\|$ - log$\|x^5 + 3\|$ + c	½ ½ 1 1
31) Correct figure $9x^2 + 4y^2 = 36$ ⇨ y = $\frac{3}{2}\sqrt{4 - x^2}$ ⇨ Area of ellipse = $4\int_0^2 ydx$ ⇨ Area = $4\int_0^2 \frac{3}{2}\sqrt{4 - x^2}dx$ = $6\int_0^2 \sqrt{4 - x^2}dx$ =$6\{\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}sin^{-1}\frac{x}{2}\}$ = 6{2 x $\pi/2$} = 6π OR Correct Figure $y^2 = 4ax$ which gives y = $2\sqrt{ax}$ Area required: = $2\int_0^a 2\sqrt{ax}$ = $4\sqrt{a}\int_0^a \sqrt{x}\, dx$ = $\frac{8a^2}{3}$	½ ½ ½ ½ 1 ½ ½ 1 + 1

32) Given x = $\sqrt{a^{sin^{-1}}t}$, y = $\sqrt{a^{cos^{-1}}t}$ Now, xy = $\sqrt{(a^{sin^{-1}}t)(a^{cos^{-1}}t)}$ =$\sqrt{a^{\pi/2}}$ Differentiating both sides w.r.t. x y+x$\frac{dy}{dx}$= 0 ⇨ $\frac{dy}{dx} = \frac{-y}{x}$	½ +½ ½ 1 ½
33)Consider f(x) = 3x^4 - 4x^3 - 12x^2 + 5 Now f'(x) = 12x^3 - 12x^2 - 24x =12(x)(x-2)(x+1) f'(x) = 0 ⇨ 12(x)(x-2)(x+1) = 0 ⇨ x=0 or x=2 or x=-1 f'(x)> 0 in (-1,0) and (2,∞) f'(x)< 0 in $(-\infty, -1)$ and (0,2) ⇨ Function is strictly increasing in (-1,0) ∪(2,∞) and strictly decreasing in $(-\infty, -1)$ ∪ (0,2)	½ ½ ½ ½ 1
34) For proving, reflexivity, Symmetry Transitivity Equivalence class[2] ={2,9} 35) y = $cos^{-1}(4x^3$ - 3x) Put x = cosθ Then y = $cos^{-1}(4cos^3\theta$ – 3cosθ) = cos^{-1}(cos3θ) = 3θ = 3cos $^{-1}$ x Differentiating both sides w.r.t x $\frac{dy}{dx} = \frac{-3}{\sqrt{1-x^2}}$	½ 1 1 ½ ½ ½ ½ ½ 1

SECTION V	
36) Max Z=7.8x+7.1y Sub to 3x+4y≤1080 3x+8y≤1920 x≤200 x≥0,y≥0 (For correct graph)	3
<table><tr><td>Corner points</td><td>Value of Z=7.8x+7.1y</td></tr><tr><td>O</td><td>0</td></tr><tr><td>A (200,0)</td><td>1560</td></tr><tr><td>B (200,120)</td><td>2412(maximum)</td></tr><tr><td>C (80,210)</td><td>2115</td></tr><tr><td>D(0,240)</td><td>1704</td></tr></table>	1
Max Z = 2412 for x = 200 and y = 120	1
OR	
i). Constraints: $2x + y \leq 60$, $x \leq 20$, $2x + 3y \leq 120$ and Non-Negativity Constraints: $x \geq 0, y \geq 0$	1½ ½
ii). Identifying Corner Points and the value at these points: (0, 0), (20, 0), (20,20) , (15, 30) and (0, 40)	2½
Maximum Value = 262.5 at (15, 30)	1
37) Consider AB = = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$	1½
= $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ =4I ⇨ $A^{-1} = \frac{1}{4}B$	1
The given system of equations is 2x – y + z = -1 -x + 2y – z = 4	

x - y + 2z = -3 This can be written as AX = C Clearly $\|A\| \neq 0$ Hence we get X = A^{-1} C ⇨ $X = \frac{1}{4} BC$ $= \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}\begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix}$ $= \frac{1}{4}\begin{bmatrix} 4 \\ 8 \\ -4 \end{bmatrix}$ $= \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$	½ ½ 1 + ½
38) Equations of planes are x + 2y + 3z = 4 2x + y – z = -5 Equation of plane through the intersection of given two planes (x + 2y + 3z) + ƛ (2x + y – z) = 4-5 ƛ ⇨ (1+2 ƛ)x + (2+ ƛ)y + (3- ƛ)z =4-5 ƛ As this plane is perpendicular to the plane -5x-3y+6z = 8 The product of their normals is zero. ⇨ (1+2 ƛ)(-5)+ (2+ ƛ)(-3) + (3- ƛ)(6)=0 ⇨ ƛ = 7/19 ⇨ The required equation of plane becomes (x + 2y + 3z) + $\frac{7}{19}$ (2x + y – z) = 4-5($\frac{7}{19}$) ⇨ 33x+45y+50z = 41 OR Equation of line along which distance to be measured: $$\frac{x+2}{3} = \frac{y-3}{2} = \frac{z+4}{5/3} = \lambda$$ General Point on line: { $3\lambda - 2$, $2\lambda + 3$, $5\lambda/3 - 4$} For point of Intersection: $\lambda = -1$ Hence, point of Intersection: $(-5, 1, -17/3)$ Required Distance = $\sqrt{110}/3$	½ 1½ ½ ½ ½ ½ 1 1 1 1 1 1

SET – B: Questions serial order in Set A are as follows:-

SET-B	1	2	3	4	5	6	7	8	9
SET-A	4	8	11	16	15	14	1	13	7

SET-B	10	11	12	13	14	15	16	17	18
SET-A	12	2	9	5	3	10	6	17	

SET-B	19	20	21	22	23	24	25	26	27	28
SET-A	20	23	27	22	26	28	19	25	21	24

SET-B	29	30	31	32	33	34	35	36	37	38
SET-A	31	33	35	29	34	30	32	38	36	37