Marking Scheme
Mathematics
Set A

Q1 (c) 1

Q2 (a) 3×3

Q3 (b) $\frac{18}{25}(3\hat{j}+4\hat{k})$

Q4 (d) -2

Q5 (a) 1

Q6 (b) $\frac{1}{4}$

Q7 (c) Transitive if (1,1) is added

Q8 (c) y^2

Q9 (c) Null Matrix

Q10 (b) 4

Q11 (c) $3\hat{a}$

Q12 (b) $\sec x + \tan x + C$

Q13 (a) 2

Q14 (d) $\frac{3}{4}$

Q15 (a) -1

Q16 (b) 6

Q17 (b) $6/7$

Q18 (b) $\sec x + \tan x$

Q19 (d)

Q20 (a)

2 marker

Q.21 $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$

$$\therefore BA = \begin{bmatrix} 2-12 & -4+6 & 6+15 \\ 4-20 & -8+10 & 12+25 \\ 2-4 & -4+2 & 6+5 \end{bmatrix}$$

$$= \begin{bmatrix} -10 & 2 & 21 \\ -16 & 2 & 37 \\ -2 & -2 & 11 \end{bmatrix} \quad \text{— (1)}$$

$\therefore b_{21} = -16$ — (½)

$b_{32} = -2$ — (½)

$\therefore b_{21} + b_{32} = -18$

Q22

$$LHL = \lim_{x\to 0^-}\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$$

$$= \lim_{h\to 0} \frac{e^{-\infty}-1}{e^{-\infty}+1} = \frac{0-1}{0+1} = -1 \quad (1)$$

$$RHL = \lim_{x\to 0^+}\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$$

$$= \lim_{h\to 0}\left(\frac{1-e^{-1/h}}{1+e^{-1/h}}\right) \quad (1)$$

$$= \frac{1-e^{-\infty}}{1+e^{-\infty}}$$

$$= 1$$

Since $LHL \neq RHL \therefore f(x)$ is not continuous at $x=0$

OR

$$LHD = \lim_{h\to 0} \frac{f(2-h) - f(2)}{-h}$$

$$= \lim_{h\to 0} \frac{(2-h)[2-h] - \{(2-1).2\}}{-h}$$

$$= \lim_{h\to 0} \frac{(2-h).1 - 2}{-h}$$

$$= \lim_{h\to 0} \frac{-h}{-h} = 1$$

$$RHD = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h\to 0} \frac{(2+h-1)\,(2+h) - (2-1)2}{h}$$

$$= \lim_{h\to 0} \frac{2+3h+h^2-2}{h} \qquad (1\tfrac{1}{2})$$

$$= \lim_{h\to 0} 3+h = 3$$

$\therefore$ LHD $\neq$ RHD so $f(x)$ is not differentiable at $x = 2$ $(\tfrac{1}{2})$

Q23 for domain $-1 \le x^2 - 4 \le 1$ (1)

$$\Rightarrow 3 \le x^2 \le 5$$

$$\Rightarrow \sqrt{3} \le x \le \sqrt{5} \text{ or}$$

$$-\sqrt{5} \le x \le -\sqrt{3} \qquad (\tfrac{1}{2})$$

$$\Rightarrow x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}] \qquad (\tfrac{1}{2})$$

24 Given $|\vec{a}|=1$, $|\vec{b}|=1$, $|\vec{c}|=1$ 1)

$\vec{a}\cdot\vec{b}=0$ and $\vec{a}\cdot\vec{c}=0$

$\Rightarrow \vec{a}\perp\vec{b}$ and $\vec{a}\perp\vec{c}$.

$\Rightarrow \vec{a}=\lambda(\vec{b}\times\vec{c})$

$\Rightarrow |\vec{a}|=|\lambda(\vec{b}\times\vec{c})|$

$\Rightarrow |\vec{a}|=|\lambda|\,|\vec{b}\times\vec{c}|$ ——— (1)

$\Rightarrow |\vec{a}|=|\lambda|\,|\vec{b}|\,|\vec{c}|\sin\theta$

$\Rightarrow 1=|\lambda|.(1).(1)\sin\frac{\pi}{6}$.

$\Rightarrow 1=|\lambda|.\frac{1}{2}$

$\Rightarrow |\lambda|=2$

$\Rightarrow \lambda=\pm 2$ — (1)

$\therefore$ equation (1) becomes

$\vec{a}=\pm 2(\vec{b}\times\vec{c})$

Q25 Given region is $\{(x,0): y=\sqrt{4-x^2}\}$ and x- axis

$y=\sqrt{4-x^2} \Rightarrow y^2=4-x^2 \Rightarrow x^2+y^2=4$ $(\frac{1}{2})$

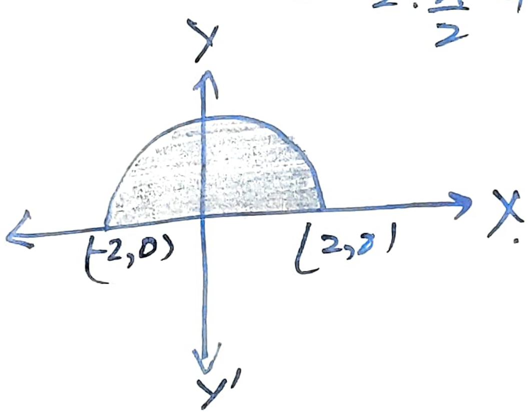
Area of shaded region

$$A = \int_{-2}^{2}\sqrt{4-x^2}\,dx \quad (1)$$

$$= \left[\frac{x}{2}\sqrt{2^2-x^2}+\frac{2^2}{2}\sin^{-1}\frac{x}{2}\right]_{-2}^{2}$$

$$= \frac{2}{2}.0+2.\frac{\pi}{2}+\frac{2}{2}.0-2\sin^{-1}(-1)$$

$$= 2.\frac{\pi}{2}+2.\frac{\pi}{2}=2\pi \text{ Squnits}$$



$(\frac{1}{2})$

OR

Q26 Given $x = a(\theta - \sin\theta)$ and $y = a(1+\cos\theta)$

$$\frac{dx}{d\theta} = a(1-\cos\theta)$$

$$\frac{dy}{d\theta} = -a\sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{-a\sin\theta}{a(1-\cos\theta)} = \frac{-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}$$ (1)

$$= -\cot\frac{\theta}{2}$$

$$\frac{d^2y}{dx^2} = -\frac{d}{dx}\left[\cot\frac{\theta}{2}\right)$$

$$= \operatorname{cosec}^2\frac{\theta}{2}\cdot\left(\frac{1}{2}\cdot\frac{d\theta}{dx}\right)$$

$$= \frac{1}{2}\operatorname{cosec}^2\theta\,\frac{\theta}{2}\cdot\frac{1}{a(1-\cos\theta)}$$

$$= \frac{1}{2a}\operatorname{cosec}^2\frac{\theta}{2}\cdot\frac{1}{2\sin^2\frac{\theta}{2}}$$

$$= \frac{1}{4a}\operatorname{cosec}^4\frac{\theta}{2}$$ $(1\frac{1}{2})$

$$\left.\frac{d^2y}{dx}\right]_{\theta=\frac{\pi}{2}} = \frac{1}{4a}\operatorname{cosec}^4\frac{\pi}{4}$$

$$= \frac{1}{4a}(\sqrt{2})^4 = \frac{1}{a}$$ $(\frac{1}{2})$

Q27 OR $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$ —— (1)

Let $y = vx$ [$\because$ (1) is a homogeneous equation]

differentiating $y = vx$ w.r.t x

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{x \cdot vx}{x^2 + (vx)^2}$$

$$v + x\frac{dv}{dx} = \frac{vx^2}{x^2 + v^2x^2} = \frac{v}{1+v^2}.$$

$$x\frac{dv}{dx} = \frac{v}{1+v^2} - v \Rightarrow x\frac{dv}{dx} = \frac{v - v - v^3}{1+v^2}$$

$$x\frac{dv}{dx} = \frac{-v^3}{1+v^2}$$

$$\frac{(1+v^2)dv}{v^3} = -\frac{dx}{x}$$ (1)

Integrating both sides

$$\int \frac{(1+v^2)dv}{v^3} = -\int \frac{dx}{x}$$

$$\Rightarrow \int \frac{dv}{v^3} + \int \frac{dv}{v} = -\log|x| + C$$

$$\Rightarrow \frac{-1}{2v^2} + \log|v| = -\log|x| + C$$

$$\Rightarrow -\frac{x^2}{2y^2} + \log\left|\frac{y}{x}\right| = -\log|x| + C$$

$$\Rightarrow \frac{-x^2}{2y^2} + \log|y| - \log|x| = -\log|x| + C$$ (1)

$$\Rightarrow \frac{-x^2}{2y^2} + \log|y| = C \text{ ——— } (2)$$

given $x = 1,\ y = 1$

$$\Rightarrow \frac{-1}{2\times 1} + \log|1| = C \Rightarrow C = \frac{-1}{2}$$

$\therefore$ (2) becomes $\frac{-x^2}{2y^2} + \log|y| = \frac{-1}{2}$

$$-\frac{x^2}{2y^2}+\log|y|=-\frac{1}{2}$$

$$\log|y|=\frac{x^2}{2y^2}-\frac{1}{2}$$

$$\log|y|=\frac{x^2-y^2}{2y^2} \quad \text{——}(3)$$

Putting $x=x_0$ and $y=e$ in (3) we get

$$\log|e|=\frac{x_0^2-e^2}{2e^2}$$

$$1=\frac{x_0^2-e^2}{2e^2} \Rightarrow x_0^2-e^2=2e^2 \qquad (1)$$

$$\Rightarrow x_0^2=3e^2 \Rightarrow x_0=\sqrt{3}e$$

or

we have $(\tan^{-1}y-x)\,dy=(1+y^2)\,dx$

$$\frac{dx}{dy}=\frac{\tan^{-1}y-x}{1+y^2}$$

$$\frac{dx}{dy}=\frac{\tan^{-1}y}{1+y^2}-\frac{x}{1+y^2}$$

$$\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{\tan^{-1}y}{1+y^2} \qquad (\tfrac{1}{2})$$

$$\frac{dx}{dy}+Px=Q$$

$$P=\frac{1}{1+y^2} \qquad Q=\frac{\tan^{-1}y}{1+y^2}$$

$$I.F=e^{\int P\,dy}$$

$$\boxed{I.F=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y}} \qquad (\tfrac{1}{2})$$

solution is

$$x.e^{\tan^{-1}y} = \int \frac{\tan^{-1}y}{1+y^2} . e^{\tan^{-1}y} dy + C$$

$$x. e^{\tan^{-1}y} = \int \frac{\tan^{-1}y}{1+y^2} . e^{\tan^{-1}y} dy + C$$

Put $\tan^{-1}y = t$

$$\therefore \frac{1}{1+y^2} dy = dt$$

$$\therefore x e^{\tan^{-1}y} = \int \underset{I}{t} \, \underset{II}{e^{t}} dt + C$$

$$x e^{\tan^{-1}y} = t e^{t} - \int e^{t} dt + C$$

$$x. e^{\tan^{-1}y} = t.e^{t} - e^{t} + C$$

$$x. e^{\tan^{-1}y} = e^{\tan^{-1}y}(\tan^{-1}y - 1) + C \quad (1\tfrac{1}{2})$$

Put $x = 0$ & $y = 0$

$$0 = e^{\tan^{-1}(0)}[\tan^{-1}(0) - 1] + C$$

$$\Rightarrow 0 = e^{0}(0-1) + C$$

$$\Rightarrow 0 = -1 + C \Rightarrow C = 1 \quad (\tfrac{1}{2})$$

$\therefore$ Solution is

$$x e^{\tan^{-1}y} = e^{\tan^{-1}y}(\tan^{-1}y - 1) + 1$$

Q28

Q28 $I = \int \frac{x^2}{x^4 - x^2 - 12}$

$= \int \frac{x^2}{(x^2-4)(x^2+3)}$ let $x^2 = t$ $(\frac{1}{2})$

$\frac{t}{(t-4)(t+3)} = \frac{A}{t-4} + \frac{B}{t+3}$

$\Rightarrow t = A(t+3) + B(t-4)$

$= (A+B)t + (3A - 4B)$

$\Rightarrow A + B = 1$

$3A - 4B = 0$

$\Rightarrow A = \frac{4}{7},\ B = \frac{3}{7}$ (1)

$\frac{x^2}{(x^2-4)(x^2+3)} = \frac{4}{7(x^2-4)} + \frac{3}{7(x^2+3)}$

$= \frac{4}{7}\int \frac{1}{x^2-4}\,dx + \frac{3}{7}\int \frac{dx}{x^2+3}\,dx$ $(1\frac{1}{2})$

$= \frac{1}{7}\log\left|\frac{x-2}{x+2}\right| + \frac{\sqrt{3}}{7}\tan^{-1}\frac{x}{\sqrt{3}} + C$

OR

$I = \int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx.$

Put $x = a\tan^2\theta$

$dx = 2a\tan\theta\sec^2\theta\,d\theta$

$\therefore I = 2a\int \sin^{-1}\left(\frac{\tan\theta}{\sec\theta}\right)\tan\theta.\sec^2\theta\,d\theta.$

$= 2a\int \underset{I}{\theta}\ \underset{II}{\tan\theta.\sec^2\theta}\,d\theta$ (1)

$$= 2a\left[\theta\int \tan\theta \sec^2\theta \, d\theta - \int\left(\frac{d}{d\theta}\cdot\theta\int \tan\theta\sec^2\theta\, d\theta\right)d\theta\right]$$

$$= 2a\left[\theta\cdot\frac{\tan^2\theta}{2} - \int\frac{\tan^2\theta}{2}\, d\theta\right]$$

$$= a\theta\tan^2\theta - a\int(\sec^2\theta - 1)\, d\theta$$

$$= a\theta\tan^2\theta - a\tan\theta + a\theta + C \quad (2)$$

$$= a\left[\frac{x}{a}\tan^{-1}\sqrt{\frac{x}{a}} - \sqrt{\frac{x}{a}} + \tan^{-1}\sqrt{\frac{x}{a}}\right] + C$$

Q29 The cartesian equations of given lines are

$x+1 = 2y = -12z$ and $x = y+2 = 6z-6$ (½)

$$\frac{x+1}{1} = \frac{y-0}{\left(\frac{1}{2}\right)} = \frac{z-0}{\left(-\frac{1}{12}\right)} \quad \text{and} \quad \frac{x-0}{1} = \frac{y+2}{1} = \frac{z-1}{\left(\frac{1}{6}\right)}$$

∴ The vector equations of given lines are

$$\vec{r} = (-\hat{i} + 0\hat{j} + 0\hat{k}) + \lambda\left(\hat{i} + \frac{1}{2}\hat{j} - \frac{1}{12}\hat{k}\right) \rightarrow \vec{a_1} + \lambda\vec{b_1}$$

and $$\vec{r} = (0\hat{i} - 2\hat{j} + \hat{k}) + \mu\left(\hat{i} + \hat{j} + \frac{1}{6}\hat{k}\right) \rightarrow \vec{a_2} + \lambda\vec{b_2}$$

$\vec{a_1} = -\hat{i} + 0\hat{j} + 0\hat{k}$ $\qquad \vec{b_1} = \hat{i} + \frac{1}{2}\hat{j} - \frac{1}{12}\hat{k}$

$\vec{a_2} = 0\hat{i} - 2\hat{j} + \hat{k}$ $\qquad \vec{b_2} = \hat{i} + \hat{j} + \frac{1}{6}\hat{k}$. (1)

here $\vec{b_1} \neq \vec{b_2}$

∴ given lines are either intersecting or skewlines

$$\therefore \; S.D = \left|\frac{(\vec{a_2} - \vec{a_1})\cdot(\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|}\right|$$

$$\vec{a_2} - \vec{a_1} = \hat{i} - 2\hat{j} + \hat{k}$$

$$\vec{b_1}\times\vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1/2 & -1/12 \\ 1 & 1 & 1/6 \end{vmatrix}$$

$$= \frac{1}{6}\hat{i} - \frac{1}{4}\hat{j} + \frac{1}{2}\hat{k}$$

$$|\vec{b_1}\times\vec{b_2}| = \frac{7}{12}$$

$$\therefore S.D = \left|\frac{(\hat{i}-2\hat{j}+\hat{k})\cdot\left(\frac{1}{6}\hat{i}-\frac{1}{4}\hat{j}+\frac{1}{2}\hat{k}\right)}{7/12}\right| \quad (1\tfrac{1}{2})$$

$$= 2 \text{ units}$$

OR

Equation of first line is

$$x = py + q, \; z = ry + s$$

i.e $\frac{x-q}{p} = y$, $\frac{z-s}{r} = y$.

i.e $\frac{x-q}{p} = y = \frac{z-s}{r}$ $(\tfrac{1}{2})$

$\therefore$ d.r's are $\langle p, 1, r \rangle$

Equation of second line is

$$x = p'y + q', \; z = r'y + s'$$

$$\frac{x-q'}{p'} = y, \; \frac{z-s'}{r'} = y \quad (1)$$

$$\frac{x-q'}{p'} = y = \frac{z-s'}{r'}$$

$\therefore$ d.r's of second line $\langle p', 1, r' \rangle$

hence the given lines are $\perp$ to each other if

$$(p)(p') + (1)(1) + (r)(r') = 0$$

i.e. $pp' + rr' + 1 = 0$. $(1\frac{1}{2})$

Q30 The function $f: R \to \{x \in R : -1 < x < 1\}$ is defined by $f(x) = \frac{x}{1+|x|}$, $x \in R$

FOR One-One: Case I :- if x is positive and y is negative

$x \neq y \Rightarrow |x| \neq |y|$ $(|x| = |y|$ if $x = -y)$

$\Rightarrow \frac{x}{1+|x|} \neq \frac{y}{1+|y|}$

$\Rightarrow f(x) \neq f(y)$, Hence 1-1

Case II : when x & y are both positive

$f(x) = f(y) \Rightarrow \frac{x}{1+x} = \frac{y}{1+y} \Rightarrow x = y$

Hence 1-1 (2)

Case III : when x & y are both negative

$f(x) = f(y) \Rightarrow \frac{x}{1-x} = \frac{y}{1-y} \Rightarrow x = y$

Hence One-One

FOR ONTO : Now let $y \in R$ such that $-1 < y < 1$

if $y > 0$ (from co-domain) then there exists $x \in R$ (domain) such that

$y = f(x) \Rightarrow y = \frac{x}{1-x}$ (1)

$y - xy = x \Rightarrow x(1+y) = y$

$\Rightarrow x = \frac{y}{1+y} \in R$ (domain)

$$f\left(\frac{y}{1+y}\right)=\frac{\left(\frac{y}{1+y}\right)}{1-\left(\frac{y}{1+y}\right)}=y$$

$\forall\ y>0$ (from codomain) then there exist $x\in R$ (domain)

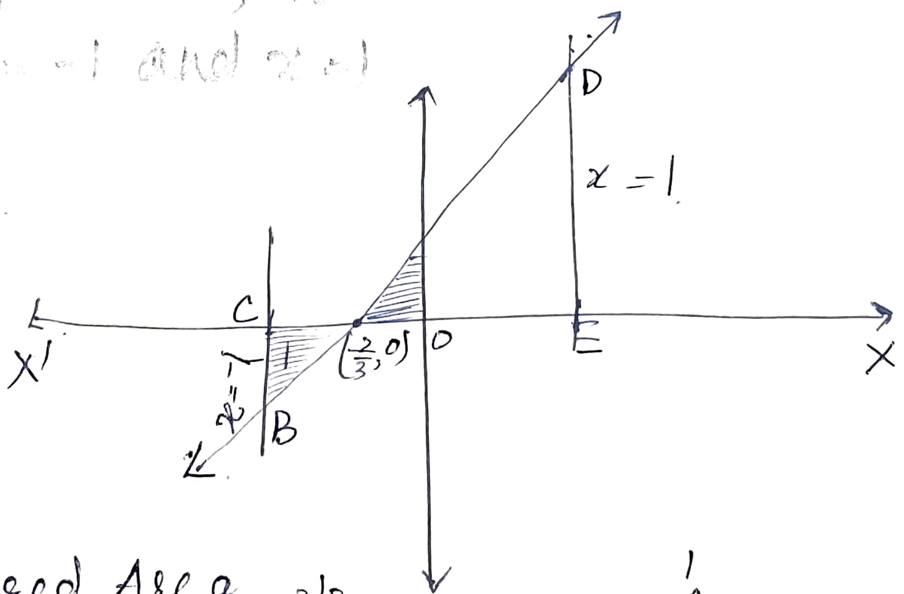
such that $y=f(x) \Rightarrow y=\frac{x}{1+x}$

$$\Rightarrow y+xy=x$$
$$\Rightarrow x(1-y)=y$$
$$\Rightarrow x=\frac{y}{1-y}$$

$$f\left(\frac{y}{1-y}\right)=\frac{\left(\frac{y}{1-y}\right)}{1+\left(\frac{y}{1-y}\right)}=y$$

$\therefore$ f is Onto, Hence f is One-One & Onto

Q.31 find the area of the region bdd by the lines $y=3x+2$, the x-axis and the ordinates $x=-1$ and $x=1$



$(1\frac{1}{2})$

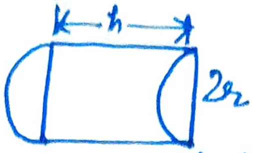
Required Area

$$=\left|\int_{-1}^{-2/3}(3x+2)dx\right|+\int_{-2/3}^{1}(3x+2)\,dx \qquad (1\tfrac{1}{2})$$

$$=\left|\left[\frac{3x^2}{2}+2x\right]_{-1}^{-\frac{2}{3}}\right|+\left[\frac{3x^2}{2}+2x\right]_{-2/3}^{1}=\frac{1}{6}+\frac{25}{6}=\frac{13}{3}$$

Q 32 let r be the radius and h be the height of half cylinder

Volume $= \frac{1}{2}\pi r^2 h = V$ (constant) — (1) ... $\frac{1}{2}$



— $\frac{1}{2}$

Total surface area of half cylinder is.

$$S = 2\left(\frac{1}{2}\pi r^2\right) + \pi r h + 2rh \quad \text{— (2)}$$

from (1) put the value of h in (2) — (1)

$$S = (\pi r^2) + \pi r\left(\frac{2V}{\pi r^2}\right) + 2r\left(\frac{2V}{\pi r^2}\right)$$

$$S = (\pi r^2) + \frac{1}{r}\left[\frac{4V}{\pi} + 2V\right]$$

$$\frac{dS}{dr} = (2\pi r) + \left(-\frac{1}{r^2}\right)\left[\frac{4V}{\pi} + 2V\right] \quad \text{— (3)}$$

for maxima / minima $\frac{dS}{dr} = 0$ (1)

$$\Rightarrow (2\pi r) + \left(\frac{-1}{r^2}\right)\left[\frac{4V}{\pi} + 2V\right] = 0$$

$$\Rightarrow 2\pi r = \left(\frac{1}{r^2}\right)\left[\frac{4V + 2V\pi}{\pi}\right]$$

$$\Rightarrow \pi r^3 = V\left[\frac{2+\pi}{\pi}\right] \quad \cancel{(1)}$$

$$\Rightarrow V = \frac{\pi^2 r^3}{\pi + 2} \quad \text{— (4)}$$

from (1) & (4)

$$\Rightarrow \frac{1}{2}\pi r^2 h = \frac{\pi^2 r^3}{\pi + 2} \quad (1)$$

$$\Rightarrow \frac{h}{2r} = \frac{\pi}{\pi + 2}$$

$\Rightarrow$ height : diameter $= \pi : \pi + 2$

differentiating (3) with respect to r

$$\frac{d^2S}{dr^2} = (2\pi) + \left(\frac{2}{r^3}\right)\left[\frac{4V}{\pi} + 2V\right] = \text{positive}$$ (as all quantities are +ve).

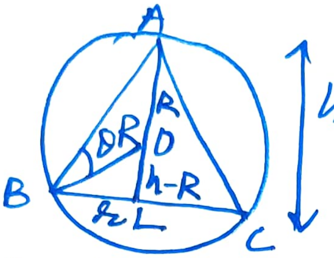
So S is minimum when height : diameter $= \pi : \pi + 2$

OR

let $2r$ be the base and h be the height of triangle which is inscribed in a circle of radius R

Area of triangle $= \frac{1}{2}(\text{base})(\text{height})$

$A = \frac{1}{2}(2r)(h) = rh$ —— ① - - - - - - - - $(\frac{1}{2})$



- - - - - - - - - - - - $(\frac{1}{2})$

Area being positive, A will be maximum or minimum if A^2 is maximum or minimum

$Z = A^2 = r^2h^2$ ———— ②

Now in triangle OLB $BL^2 = OB^2 - OL^2$

In ΔOBD

$Z = A^2 = r^2h^2$ $r^2 = R^2 - (h-R)^2 \Rightarrow r^2 = 2hR - h^2$

put in ②

$Z = h^2(2hR - h^2)$ (1)

$\Rightarrow Z = (2h^3R - h^4)$

$\Rightarrow \frac{dZ}{dh} = 6h^2R - 4h^3$ ———— ③

for maxima/minima $\frac{dZ}{dh} = 0$ - - - - - - $(\frac{1}{2})$

$\Rightarrow 6h^2R - 4h^3 = 0$

$\Rightarrow 6R = 4h \; (h \neq 0)$

$\Rightarrow h = \frac{3R}{2}$ - - - - - - - - - - - - (1)

differentiating ③ w.r.t h

$\Rightarrow \frac{d^2Z}{dh^2} = 12hR - 12h^2$

$\Rightarrow \left.\frac{d^2Z}{dh^2}\right]_{h=\frac{3R}{2}} = 12\left(\frac{3R}{2}\right)R - 12\left(\frac{3R}{2}\right)^2$

$= 18R^2 - 27R^2 = -ve$

So $Z = A^2$ is maximum when $h = \frac{3R}{2}$ - - - - $(\frac{1}{2})$

when $h = \frac{3R}{2}$, $r^2 = 2hR - h^2$

$$= 2R \cdot \frac{3R}{2} - \left(\frac{3R}{2}\right)^2$$

$$r^2 = \frac{3R^2}{4}$$

$$r = \frac{\sqrt{3}}{2} R$$

$$\tan\theta = \frac{h}{r} = \frac{\frac{3R}{2}}{\frac{\sqrt{3}R}{2}} = \sqrt{3} \quad \theta = \frac{\pi}{3} \quad \text{---} \quad (1)$$

triangle ABC is equilateral Δ

Q33 $$I = \int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x}\, dx \quad \text{---} \quad (1)$$

Using the property $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$$I = \int_0^{\pi/2} \frac{\left(\frac{\pi}{2} - x\right) \sin\left(\frac{\pi}{2} - x\right) \cos\left(\frac{\pi}{2} - x\right)}{\sin^4\left(\frac{\pi}{2} - x\right) + \cos^4\left(\frac{\pi}{2} - x\right)}$$

$$I = \int_0^{\pi/2} \frac{\left(\frac{\pi}{2} - x\right) \cos x \sin x}{\cos^4 x + \sin^4 x}\, dx \quad \text{---} \quad (2)$$ (1)

Adding (1) & (2)

$$2I = \int_0^{\pi/2} \frac{\left(\frac{\pi}{2} \cdot \sin x \cos x\right)}{\sin^4 x + \cos^4 x}\, dx$$

$$I = \frac{\pi}{4} \int_0^{\pi/2} \left(\frac{\sin x \cos x}{\sin^4 x + \cos^4 x}\right) dx.$$

$$I = \frac{\pi}{4} \int_0^{\pi/2} \left(\frac{\sin x \cos x / \cos^4 x}{\frac{\sin^4 x}{\cos^4 x} + 1}\right) dx$$ $\left(1\frac{1}{2}\right)$

$$I = \frac{\pi}{4}\int_0^{\pi/2} \frac{\tan x \sec^2 x}{\tan^4 x + 1}\,dx$$

Put $\tan^2 x = z$

$\therefore 2\tan x \sec^2 x\, dx = dz$

$\Rightarrow \tan x \sec^2 x\, dx = \frac{dz}{2}$

when $x = 0$, $z = 0$ when $x = \frac{\pi}{2}$, $z = \infty$

$$\therefore I = \frac{\pi}{4}\int_0^{\infty} \frac{\frac{dz}{2}}{z^2+1}$$

$$I = \frac{\pi}{8}\int_0^{\infty}\frac{dz}{1+z^2}$$

$$= \frac{\pi}{8}\left[\tan^{-1} z\right]_0^{\infty} \qquad (1\tfrac{1}{2})$$

$$= \frac{\pi}{8}\left[\tan^{-1}\infty - \tan^{-1} 0\right]$$

$$= \frac{\pi}{8}\left(\frac{\pi}{2} - 0\right) \qquad (\tfrac{1}{2})$$

$$= \frac{\pi^2}{16}$$

OR

$$\text{let} \quad I = \int_{-1}^{1} \frac{x^3 + |x| + 1}{x^2 + 2|x| + 1}\,dx$$

$$= \int_{-1}^{1} \frac{x^3}{x^2+2|x|+1}\,dx + \int_{-1}^{1}\frac{|x|+1}{x^2+2|x|+1}\,dx$$

$$= I_1 + I_2 \qquad (1)$$

now $I_1 = \int_{-1}^{1} \frac{x^3}{x^2+2|x|+1}\,dx$

Assume $f(x) = \frac{x^3}{x^2+2|x|+1}$

then $f(-x) = \frac{(-x)^3}{(-x)^2+2|-x|+1} = -\frac{x^3}{x^2+2|x|+1}$

$= -f(x)$ $(1\frac{1}{2})$

$\therefore I_1 = 0$

$I_2 = \int_{-1}^{1} \frac{|x|+1}{x^2+2|x|+1}\,dx$

Assume $f(x) = \frac{|x|+1}{x^2+2|x|+1}$

Then $f(-x) = \frac{|-x|+1}{(-x)^2+2|-x|+1} = \frac{|x|+1}{x^2+2|x|+1} = f(x)$

$\therefore I_2 = 2\int_{0}^{1} \frac{|x|+1}{x^2+2|x|+1}\,dx.$

$= 2\int_{0}^{1} \frac{x+1}{x^2+2x+1}\,dx$

$= 2\int_{0}^{1} \frac{x+1}{(x+1)^2}\,dx$

$= 2\int_{0}^{1} \frac{1}{x+1}\,dx$ $(2\frac{1}{2})$

$= 2\left[\log|x+1|\right]_0^1$

$= 2\left[\log 2 - 0\right]$

$= 2\log 2$

Q34 Let E : the change doesn't take place
Let E_1 : person A is appointed
E_2 : person B is appointed (1½)
E_3 : person C is appointed
Clearly we have $P(E_1) = \frac{1}{7}$

$P(E_2) = \frac{2}{7}$

$P(E_3) = \frac{4}{7}$

$P\left(\frac{E}{E_1}\right) = \frac{2}{10}$

$P\left(\frac{E}{E_2}\right) = \frac{5}{10}$ (1)

$P\left(\frac{E}{E_3}\right) = \frac{7}{10}$

By Baye's Theorem $P(E_1/E) = \dfrac{P(E/E_1)\,P(E_1)}{P\left(\frac{E}{E_1}\right)P(E_1) + P\left(\frac{E}{E_2}\right)P(E_2) + P\left(\frac{E}{E_3}\right)P(E_3)}$

$$P\left(\frac{E_1}{E}\right) = \frac{\frac{2}{10} \times \frac{1}{7}}{\frac{2}{10} \times \frac{1}{7} + \frac{5}{10} \times \frac{2}{7} + \frac{7}{10} \times \frac{4}{7}}$$

$$P\left(\frac{E_1}{E}\right) = \frac{2}{2+10+28} \quad (2\tfrac{1}{2})$$

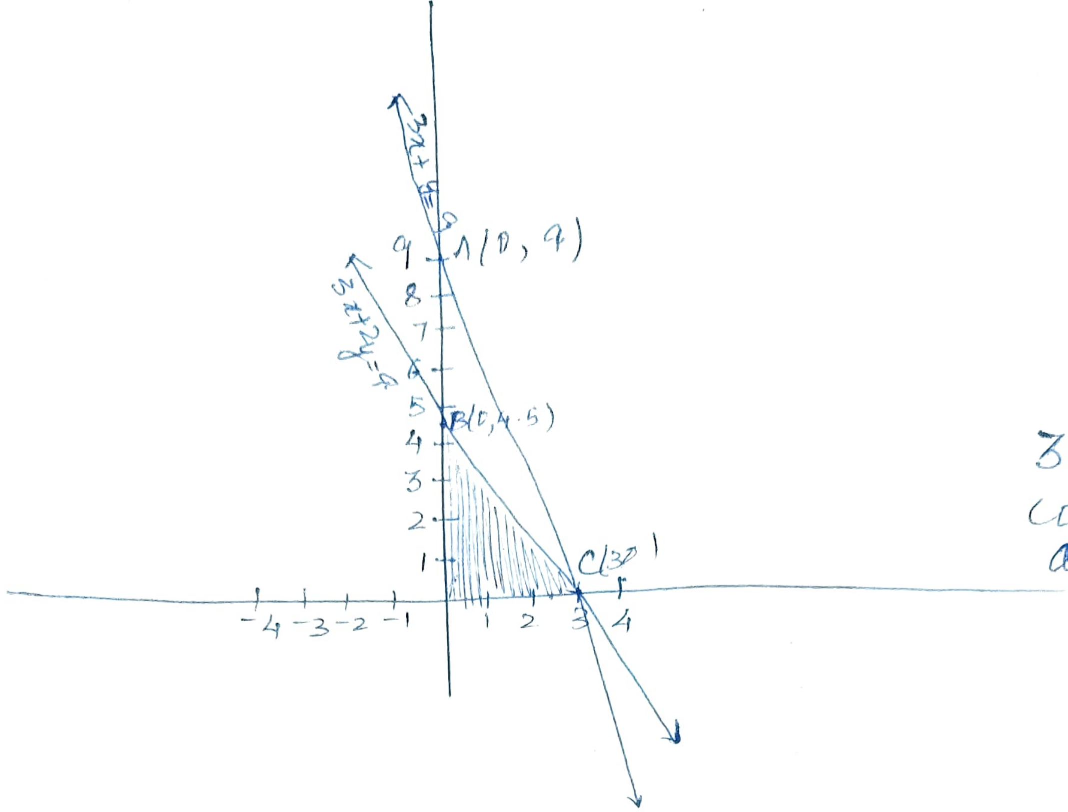
$$P\left(\frac{E}{E_1}\right) = \frac{2}{40} \text{ or } \frac{1}{20}.$$

Q35

Corner points	Value of P.
O(0, 0)	0
B(0, 4.5)	180
C(3, 0)	210 → Max value

(1½)

Maximum value of $P = 210$ at $x = 3$ & $y = 0$ (½)
Correct figure and shading ———— (3)



3marks for correct figure and shading

Corner pts.	Value of P
O (0,0)	0
B (0,4.5)	180
C (3, 0)	210 → Max Valu

$1\frac{1}{2}$

Maximum value of $P = 210$ at $x = 3$ & $y = 0$ $\frac{1}{2}$

Q36. (1) Matrix for the new mix

$$= \begin{bmatrix} 5 & 1 & 1 \\ 6.5 & 2.5 & 0.5 \\ 4.5 & 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ -0.5 & 0.5 & 0.5 \\ 0.5 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 2 & 1 \\ 6 & 3 & 1 \\ 5 & 3 & 2 \end{bmatrix}$$ (1½)

(2) $$\begin{bmatrix} 50 & 30 & 20 \end{bmatrix} \begin{bmatrix} 5 & 2 & 1 \\ 6 & 3 & 1 \\ 5 & 3 & 2 \end{bmatrix} = \begin{bmatrix} 250+180+100, & 100+90+60 & 50+30+40 \end{bmatrix}$$

$$= \begin{bmatrix} 530 & 250 & 120 \end{bmatrix}$$ (1½)

530 Kg of Flour, 250 Kg of Fat & 120 Kg of Sugar.

(3) $$\begin{bmatrix} 5 & 6 & 5 \\ 2 & 3 & 3 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 370 \\ 170 \\ 80 \end{bmatrix}$$ as $5x_1 + 6x_2 + 5x_3 = 370$

$2x_1 + 3x_2 + 3x_3 = 170$

$x_1 + x_2 + 2x_3 = 80.$

$AX = B \Rightarrow X = A^{-1}B$

$|A| = 4 \neq 0$

$$\text{Adj} A = \begin{bmatrix} 3 & -7 & 3 \\ -1 & 5 & -5 \\ -1 & 1 & 3 \end{bmatrix}$$ (2)

$$A^{-1} = \frac{\text{adj} A}{|A|} = \frac{1}{4} \begin{bmatrix} 3 & -7 & 3 \\ -1 & 5 & -5 \\ 1 & 1 & 3 \end{bmatrix}$$

$$X = \frac{1}{4} \begin{bmatrix} 3 & -7 & 3 \\ -1 & 5 & -5 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 370 \\ 170 \\ 80 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 160 \\ 80 \\ 40 \end{bmatrix} = \begin{bmatrix} 40 \\ 20 \\ 10 \end{bmatrix}$$

Hence production of products A, B & C are 40 units, 20 units & 10 units resp.

Q37. $\frac{dp}{dx} = \frac{-135x}{\sqrt{9+x^2}}$

(I) $\Rightarrow p(x) = \int \frac{dp}{dx}\,dx$

$= \int \frac{-135x}{\sqrt{9+x^2}}\,dx$

Put $9+x^2 = t$

$2x\,dx = dt$

$p(x) = \int -\frac{135}{2}(t)^{-1/2}\,dt$

$= -\frac{135}{2} \times \frac{t^{1/2}}{1/2} + c$

$= -135\,t^{1/2} + c$

$= -135\sqrt{9+x^2} + c$

Since $p = 30$ when $x = 4$ we have

$30 = -135\sqrt{9+4^2} + c$ (2)

$\Rightarrow c = 30 + 135 \times 5 = 705$

$\Rightarrow p(x) = -135\sqrt{9+x^2} + 705$

(II) When 300 units are demanded

$\Rightarrow x = 3$

$p(3) = -135\sqrt{9+9} + 705$

$= -135\sqrt{18} + 705$ (1)

$= 132.24$ \$ per unit.

When $x = 0$

$p(0) = -135\sqrt{9} + 705$

$= \$300$ per unit.

(III). When $p(x) = 20$

$20 = -135\sqrt{9+x^2} + 705$

$135\sqrt{9+x^2} = 685$ (1)

$\sqrt{9+x^2} = \frac{685}{135}$

$9+x^2 = 25.75$

$x^2 = 16.75$

$x = 4.09$.

$\Rightarrow$ 409 units will be demanded

Q38 (I) $\vec{A}$ = Pr of P_2 − Pr of P_1

$= (21\hat{i} + 8\hat{j} + 4\hat{k}) - (6\hat{i} + 8\hat{j} + 4\hat{k})$

$= 15\hat{i} + 0\hat{j} + 0\hat{k}$

Components of A are 15, 0, 0

$\vec{B}$ = Pr of P_4 − Pr of P_1

$= (6\hat{i} + 16\hat{j} + 10\hat{k}) - (6\hat{i} + 8\hat{j} + 4\hat{k})$

$= 0\hat{i} + 8\hat{j} + 6\hat{k}$ (1)

Components of B are (0, 8, 6)

(II) $\vec{N} = \vec{A} \times \vec{B}$ as $\vec{A} \times \vec{B}$ is ⊥ to both $\vec{A}$ & $\vec{B}$

$$= \begin{vmatrix} i & j & k \\ 15 & 0 & 0 \\ 0 & 8 & 6 \end{vmatrix} = 0\hat{i} - 90\hat{j} + 120\hat{k}$$ (1½)

Components of $\vec{N}$ are 0, −90, 120

(III) $|\vec{N}| = \sqrt{0 + (-90)^2 + (120)^2} = \sqrt{22500} = 150$ units

$|\vec{S}| = 1$ as $\vec{S}$ is a unit vector.

$\vec{F} = 910\,\hat{S} = 910\left(\frac{1}{2}\hat{i} - \frac{6}{7}\hat{j} + \frac{1}{7}\hat{k}\right)$

$= 455\hat{i} - 780\hat{j} + 130\hat{k}$

(1½)

& $|\vec{F}| = 910$

$\vec{F} \cdot \vec{N} = (455\hat{i} - 780\hat{j} + 130\hat{k})(0\hat{i} - 90\hat{j} + 120\hat{k})$

$= 0 + 780 \times 90 + 130 \times 120$

$= 70200 + 15600 = 85800$ Watts.