Marking Scheme

Class XII

Maths

Q1 (b) $\pi/6$

Q2 (b) $x+2\cos x+c$

Q3 (c) $xy=c$

Q4 (b) 2

Q5 (c) $(-\infty,0)$

Q6 (d) Neither one-one nor onto

Q7 (b) $[1,2]$

Q8 (d) $\frac{x}{\sqrt{1-x^2}}$

Q9 (a) A

Q10 (a) $P(x)P(y)=P(x+y)$

Q11 (d) 8 or -8

Q12 (a) Continous and differentiable
at $x = 0$

$\frac{dy}{dx}$

Q13 (c) $\dfrac{(1+\log y)^2}{\log y}$

Q14 (b) Absolute Min value $= -1$
Absolute Max value $= \sqrt{2}$

Q15 (a) $-\frac{2}{\pi}$

Q16 (a) $a(\cos t + t\sin t)$

Q17 - (b) $-\pi/3$

Q18 (b) $e^x \sec x + C$

Q19 (c)

Q20 (d)

Q21
$$y = x^{1/x}$$
$$\log y = \frac{1}{x}\log x \quad \text{—— } (1/2)$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2} + \log x\left(-\frac{1}{x^2}\right) \quad \text{—— } (1/2)$$

$$\frac{dy}{dx} = y\left[\frac{1}{x^2} + \log x\left(\frac{-1}{x^2}\right)\right] \text{ — } (1/2)$$

$$= \frac{x^{1/x}}{x^2}[1-\log x] = 0 \text{ — } (1/2)$$

~~Q22~~ OR

$$y = x^x - 2^{\sin x}$$

$$u = x^x$$

$$\log u = x \log x$$

$$\frac{1}{u}\frac{du}{dx} = x\cdot\frac{1}{x} + \log x$$

$$\frac{du}{dx} = u[1+\log x] = x^x(1+\log x)$$

$$\frac{dy}{dx} = x^x(1+\log x) - 2^{\sin x}\log 2 \cos x$$

Q22 $x\sqrt{1+y} + y\sqrt{1+x} = 0$

$$\left.\begin{aligned} x\sqrt{1+y} &= y\sqrt{1+x} \\ x^2(1+y) &= y^2(1+x) \\ x^2-y^2 &= y^2x - x^2y \\ x^2-y^2 &= xy(y-x) \end{aligned}\right] \text{ — } (1/2)$$

$x+y=-xy$ —— $(1/2)$

$1+\frac{dy}{dx} = -y - x\frac{dy}{dx}$ —— $(1/2)$

$\frac{dy}{dx} = \frac{-1}{(1+x)^2}$ —— $(1/2)$

Q23 $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$ —— $(1/2)$

$A^2 + aA + bI = 0$

$\begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + a\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ —— $(1/2)$

$11+3a+b=0$
$4+a=0$ $\quad a=-4$
$b=1$ —— (1)

Q24 – $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right] + \tan^{-1}$

$\tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right] + \pi/4$ —— $(1/2)$

$\tan^{-1}\left[2\cos\pi/3\right] = \tan^{-1}\left[2\times\frac{1}{2}\right] + \pi/4$ —— $(1/2)$

—— $(1/2)$ $= \pi/4 + \pi/4$ —— $(1/2)$

$= \pi/2$

$y\ dx$

25 - $\displaystyle\int \frac{\sin x}{\sin(x+a)}\,dx$

$\displaystyle\int \frac{\sin(\overline{x+a}-a)}{\sin(x+a)}\,dx$ — (1/2)

$\displaystyle\int \frac{\sin(x+a)\cos a - \cos(x+a)\sin a}{\sin(x+a)}\,dx$ — (1/2)

$\displaystyle\int \cos a - \int \sin a \cos(x+a)\,dx$ — (1/2)

$x\cos a - \sin a \sin(x+a) + C$ — (1/2)

OR

$\displaystyle\int \sin x \sin 2x \sin 3x\,dx$

$\displaystyle\int -[\cos 3x - \cos x] \sin 3x\,dx$ — (1/2)

$\displaystyle\frac{1}{2}\int -[\cos 3x$

$\displaystyle -\frac{1}{2}\left[\int \cos 3x \sin x + \int \cos x \sin 3x\,dx\right]$ — (1/2)

$\displaystyle -\frac{1}{4}\int \sin 4x - \sin 2x\,dx + \frac{1}{4}\int \sin 4x + \sin 2x\,dx.$ — (1/2)

$\displaystyle \frac{1}{4}\int \sin 2x - \sin 6x + \sin 4x\,dx$

$\displaystyle -\frac{\cos 2x}{8} + \frac{\cos 6x}{24} - \frac{\cos 4x}{16} + c$ — (1/2)

Q26 - $f(x) = \frac{x}{2} + \frac{2}{x}$

$f'(x) = \frac{1}{2} - \frac{2}{x^2}$ ——— (1/2)

For max or min $f'(x) = 0$ —— (1/2)

$x = \pm 2$ ——— (1/2) + (1/2)

$f''(x) = \frac{4}{x^3} > 0$ for $x = 2$ ——— (1/2)

So f has local min. at $x = 2$ — (1/2)

Q27 - $\int x^2 \log(1+x^2)\, dx$

$\log(1+x^2)\frac{x^3}{3} - \int \frac{2x}{(1+x^2)} \times \frac{x^3}{3}\, dx$ ——— (1/2)

$\frac{x^3 \log(1+x^2)}{3} - \frac{2}{3}\int \frac{x^4}{1+x^2}\, dx$ ——— (1/2)

$\frac{x^3}{3}\log(1+x^2) - \frac{2}{3}\int \frac{x^4 - 1 + 1}{1+x^2}\, dx$ ——— (1/2)

$\frac{x^3}{3}\log(1+x^2) - \frac{2}{3}\int (x^2-1) + \frac{1}{1+x^2}\, dx$ ——— (1/2)

$\frac{x^3}{3}\log(1+x^2) - \frac{2}{3}\left[\frac{x^3}{3} - x + \tan^{-1}x\right] + c$ ——— (1)

OR

$$\int \frac{\cos x}{\sin 3x}\,dx$$

$$\int \frac{\cos x}{3\sin x - 4\sin^3 x}\,dx \quad \text{---} (1/2)$$

$\sin x = t$
$\cos x\, dx = dt$

$$\int \frac{\cancel{\cos x}\ \cancel{\sin x}}{\cancel{3\sin^2 x} -}$$

$$\int \frac{dt}{3t-4t^3} \quad \text{---} (1/2)$$

$$\int \frac{dt}{t(3-4t^2)}$$

Let $\dfrac{1}{t(3-4t^2)} = \dfrac{A}{t} + \dfrac{Bt+C}{3-4t^2}$ --- (1/2)

$1 = A(3-4t^2) + (Bt+C)\,t$

$3A = 1$ $\quad -4A + B = 0$

$A = 1/3$ $\quad -\frac{4}{3} + B = 0 \quad B = 4/3$ --- (1/2)

$$\int \frac{dt}{t(3-4t^2)} = \int \frac{1/3}{t} + \int \frac{4/3}{3-4t^2} \quad \text{---} (1/2)$$

$$\frac{1}{3}\log t + \frac{1}{4}\times\frac{4}{3}\int \frac{dt}{\frac{3}{4}-t^2}$$

Q28

$$\int \frac{5x}{(x+1)(x^2-4)}\,dx$$

Let $$\frac{5x}{(x+1)(x^2-4)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+2}$$ — (1/2)

$$5x = A(x^2-4) + B(x+2)(x+1) + C(x+1)(x-2)$$ — (1/2)

$$\left.\begin{aligned} B+A+C &= 0 \\ -4A+2B-2C &= 0 \\ B-C &= 5 \end{aligned}\right]$$ — (1/2)

$$A = \frac{5}{3}, \quad B = \frac{5}{6}, \quad C = -5/2$$ — (1/2)

$$\int \frac{5x}{(x+1)(x^2-4)} = \frac{5}{3}\int\frac{dx}{x+1} + \frac{5}{6}\int\frac{dx}{x-2} - \frac{5}{2}\int\frac{dx}{x+2}$$ — (1/2)

$$= \frac{5}{3}\log|x+1| - \frac{5}{2}\log|x+2| + \frac{5}{6}\log|x-2| + C$$ — (1/2)

Q29 — Let $u = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, $v = \tan^{-1} x$.

Let $x = \tan\theta$

$$u = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right) \text{ — } (1/2)$$

$$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right) = \tan^{-1}\left(\tan\theta/2\right) \text{ — } (1/2)$$

$$= \theta/2 = \frac{\tan^{-1} x}{2} \text{ — } (1/2)$$

$$\frac{du}{dx} = \frac{1}{2(1+x^2)} \text{ — } (1/2)$$

$$v = \tan^{-1} x$$

$$\frac{dv}{dx} = \frac{1}{1+x^2} \text{ — } (1/2)$$

$$\frac{du}{dv} = \frac{1}{2} \text{ — } (1/2)$$

Q30 — $$\frac{dy}{dx} + \frac{2x}{1+x^2}\cdot y = \frac{1}{(1+x^2)^2}$$

$$\frac{dy}{dx} + Py = Q$$

$$P = \frac{2x}{1+x^2} \qquad Q = \frac{1}{(1+x^2)^2} \text{ — } (1/2)$$

$$I.F. = e^{\int P\,dx} = e^{\int \frac{2x}{1+x^2}dx} = e^{\log t} = 1+x^2 \text{ — } (1/2)$$

Solⁿ :

$$y(1+x^2) = \int \frac{1}{(1+x^2)^2}(1+x^2)\,dx \quad \text{---} (1/2)$$

$$y(1+x^2) = \int \frac{dx}{1+x^2} + C \quad \text{---} (1/2)$$

$$y(1+x^2) = \tan^{-1}x + C$$

$$C = -\pi/4 \quad \text{---} (1/2)$$

So particular solution is

$$y(1+x^2) = \tan^{-1}x - \pi/4 \quad \text{---} (1/2)$$

Q31-a)

$$\begin{vmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{vmatrix} = 0 \quad \text{---} (1/2)$$

$$x(x^2-3x-49) = 0 \quad \text{---} (1/2)$$

$$x = 0,\ x = \frac{1}{2}(3 \pm \sqrt{205}) \quad \text{---} (1/2)$$

b)

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}}_{(1/2)} \underbrace{\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix}}_{(1/2)} = \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}_{(1/2)}$$

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$$\left.\begin{aligned} -3x-5y+4z &= 6000 \\ -x+3y-z &= 5000 \\ x-4y-6z &= 13000 \end{aligned}\right] \text{— (1)}$$

$$\begin{bmatrix} -3 & -5 & 4 \\ -1 & 3 & -1 \\ 1 & -4 & -6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6000 \\ 5000 \\ 13000 \end{bmatrix}$$

$x = 3000,\ y = 1000,\ z = 2000$ — (1)

$|A|$ — $\cancel{(1/2)}$ (1)

Adj A — (2)

OR

— (1/2)

$|A| = 10$

$$A^{-1} = \frac{1}{16}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \text{ — (2)}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$B \quad X = C$ — (1/2)

$$X = B^{-1} C$$
$$= (A')^{-1} C$$
$$X = (A^{-1})' C$$ — (1)

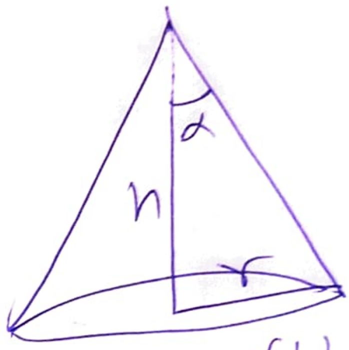
$$x = 9/5,\ y = 2/5,\ z = 7/5$$ — (1)

Q34–

Q33– $\tan^{-1}\alpha = 0.5$ — 1/2

$$\frac{r}{h} = 0.5$$

$$r = h/2$$ — (1/2)



— (1)

$$V = \frac{1}{3}\pi r^2 h$$ — (1/2)

$$V = \frac{\pi h^3}{12}$$ — (1/2)

$$\frac{dv}{dt} = \frac{\pi h^2}{4}\frac{dh}{dt}$$ — (1/2)

So $\frac{dh}{dt} = \frac{5}{4\pi} = 0.398$ m/h — (1)

(1/2)

Q34 - Let $f(x) = f(y)$

$x^3 + x = y^3 + y$ —— (1)

$x^3 - y^3 = y - x$

—— (1/2)

$x - y = 0$ —— (1/2)

$x = y$

Onto Let $y = f(x) = x^3 + x$ —— (1/2)

For every real value of y, the eqn.

$x^3 + x - y = 0$ has a real root α —— (1)

such that $\alpha^3 + \alpha - y = 0$ —— (1/2)

So $y = f(\alpha)$ —— (1/2)

Hence f is onto —— (1/2)

OR

Reflexive

For every $(a, b) \in R$

$a + b = b + a$ —— (1/2)

So $(a, b) \, R \, (a, b)$ —— (1/2)

Hence R is reflexive

Symmetric

Let $(a,b) R (c,d)$

$\Rightarrow a+d = b+c$ ——— (1/2)

so $d+a = c+b$

$\Rightarrow (c,d) R (a,b)$ ——— (1/2)

Hence R is symmetric

Transitive

Let $(a,b) R (c,d)$ and $(c,d) R (e,f)$

$\Rightarrow a+d = b+c$ and $c+f = d+e$ (1/2)

So $a+f = b+e$ ——— (1/2)

Hence $(a,b) R (e,f)$

So R is transitive and hence R is an equivalence relation ——— (1/2)

Q36 - Capacity = area × depth

$= x^2 h = 250$

$\Rightarrow x^2 = \frac{250}{h}$ ——— (1/2)

C (cost) $= 500x^2 + 4000h^2$

$C = 500\left(\frac{250}{h}\right) + 4000h^2$

$= \frac{125000}{h} + 4000h^2$ ——— (1/2)

$$\frac{dc}{dh} = \frac{-125000}{h^2} + 8000h \quad \text{—— } (1/2)$$

$$\frac{dc}{dh} = 0$$

$$\Rightarrow \frac{-125000}{h^2} + 8000h$$

$$\frac{dc}{dh} = 0 \quad \text{—— } (1/2)$$

$$\Rightarrow h = \frac{5}{2}\, m \text{ or } 2.5\, m$$

iii) a) $\frac{d^2c}{dh^2} = \frac{250000}{h^3} + 8000$ —— (1/2)

So C is min —— (1/2)

$$\left.\frac{d^2c}{dh^2}\right|_{h=2.5\,m} > 0$$

—— (1)

Min cost = C = Rs 75000

OR

$h = \frac{5}{2}\, m$ when $\frac{dc}{dh} = 0$

For h less than $\frac{5}{2}$ $\frac{dc}{dh} < 0$ —— (1/2)

—— (1/2)

For $h > \frac{5}{2}$, $\frac{dc}{dh} > 0$ so by first derivative test min at $h = 5/2$

$x^2 = \frac{250}{4}$

$\Rightarrow x = 10m$] — (1)

Also $x = 44$

Q37 — i) No. of relations $= 2^6 = 64$ — (1)

ii) No. of possible functions $= 2^3 = 8$ — (1)

iii) R is an equivalence relation as it is reflexive, symmetric and transitive.

OR

Since f is not a 1-1 function

$\therefore$ f is not ~~bijective~~ bijective

Q38 a) $|x-1| = \begin{cases} x-1, & x \geq 1 \\ -(x-1), & x < 1 \end{cases}$ — (1/2)

$LHD = \lim_{h\to 0} \frac{f(1) - f(1-h)}{h}$

$= -1$ — (1/2)

$RHD = \lim_{h\to 0} \frac{f(1) - f(1+h)}{h}$

$= 1$ — (1/2)

So $LHD \neq RHD$

f is not differentiable at $x = 1$ — (1/2) since x^2 and $\cos x$ are continous functions and composition of two continous functions is continous so $f(x) = \cos x^2$ is continous

Q35 – $\frac{dy}{dx} = \frac{y\cos\left(\frac{y}{x}\right) + x}{x\cos\left(\frac{y}{x}\right)}$

$y = vx$

$v + x\frac{dv}{dx} = \frac{v\cos v + 1}{\cos v}$

$x\frac{dv}{dx} = \frac{v\cos v + 1}{\cos v} - v$

$\int \cos v\, dv = \int \frac{dx}{x}$

$\sin v = \log x + \log C$

$\sin\frac{y}{x} = \log|Cx|$